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EFFICIENT FAMILY OF EXPONENTIAL AND DUAL ESTIMATORS OF FINITE POPULATION MEAN IN RANKED SET SAMPLING

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ABSTRACT

This study proposed improved family of exponential estimators and dual type ratio estimator of finite population mean using some known population parameters of the auxiliary variable in Ranked Set Sampling (RSS). It has been shown that this method is highly beneficial to the estimation based on Simple Random Sampling (SRS). The bias and mean squared error of the proposed estimators with first degree approximation are derived. Theoretically, it is shown that the suggested estimators are more efficient than the estimators in simple random sampling. It is also shown that the suggested dual estimator is more efficient than the usual ratio estimator in Ranked set sampling.

KEYWORDS: *Exponential Estimators, Dual Estimator, Ratio Estimator, Ranked Set Sampling, Population Mean, Auxiliary Variable, Bias, Mean Squared Error*

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1. INTRODUCTION

The literature on Ranked set sampling describes a great variety of techniques for using auxiliary information to obtain more efficient estimators. Ranked set sampling was first suggested by McIntyre (1952) to increase the efficiency of estimator of population mean. Kadilar et al. (2009) used this technique to improve ratio estimator given by Prasad (1989). Mehta and Mandowara (2013) suggested a modified ratio-cum-product estimator of finite population mean using ranked set sampling. Here, we propose improved exponential family of ratio type estimators and dual estimator for the population mean using some known parameters of the auxiliary variable in ranked set sampling.

Let $U=\{U_1,U_2,.........,U_N\}$ be the finite population of size and let y and , respectively, be the study and auxiliary variables. A sample of size n is drawn, using simple random sampling without replacement, to estimate the population mean $\bar{Y}=\frac{1}{N}\sum_{i=1}^{N}y_i$ of study variable y.

The classical ratio estimator given by Cochran (1940) for estimating the population mean , respectively for SRS, is defined as

$$\bar{y}_R = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right) \tag{1.1}$$

Bahl and Tuteja(1991) was the first to suggest an exponential ratio type estimator as

$$t_1 = \bar{y}\exp\left[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right] \tag{1.2}$$

Following Kadilar and Cingi (2006) and Khoshnevisan (2007), Singh Rajesh et al.(2007) define modified exponential estimator for estimating $\bar{Y}$ as

$$\bar{y}_e = \bar{y}\exp\left[\frac{(a\bar{X}+b)-(a\bar{x}+b)}{(a\bar{X}+b)+(a\bar{x}+b)}\right] \tag{1.3}$$

Where $a(\neq 0), b$ are either real numbers or the functions of the known parameters of the auxiliary variable x such as coefficient of variation (C_x) and coefficient of kurtosis $\beta_2(x)$ and correlation coefficient (ρ).

Srivenkataramana and Tarcy (1980) considered the following dual estimator of the population mean $\bar{Y}$ based on the use of the mean value of the non-sampled information of the auxiliary variable defined as

$$\bar{y}_{nsu} = \bar{y}\left[\frac{N\bar{X}-n\bar{x}}{(N-n)\bar{X}}\right] \tag{1.4}$$

Or $\bar{y}_{nsu} = \bar{y}\left[\frac{\bar{x}^*}{\bar{X}}\right]$

where $\bar{x} = \frac{N\bar{X}-n\bar{x}}{(N-n)} = \frac{1}{N-n}\sum_{i=1}^{N-n} x_i$ denotes the mean of non sampled units of the auxiliary variable. $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ and $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ are sample means of *y* and x respectively based on sample size *n*. Here, it is assumed that $\bar{X} = \frac{1}{N}\sum_{i=1}^{N} x_i$, population mean of auxiliary variable, is known.

To the first degree of approximation, the bias and mean squared error (MSE) of the estimators $\bar{y}_e$ and $\bar{y}_{nsu}$ are given as

$$B(\bar{y}_e) = \gamma\bar{Y}(\theta^2 C_x^2 + \theta\rho C_x C_y) \tag{1.5}$$

$$MSE(\bar{y}_e) = \gamma\bar{Y}^2\left[C_y^2 + \theta^2 C_x^2 - 2\theta\rho_{yx} C_y C_x\right] \tag{1.6}$$

and

$$B(\bar{y}_{nsu}) = E(\bar{y}_{nsu}) - \bar{Y} = -\frac{\bar{y}\rho_{xy}C_x C_y}{N} = -\frac{S_{xy}}{N\bar{X}} \tag{1.7}$$

$$MSE(\bar{y}_{nsu}) = \gamma\bar{Y}^2\left[C_y^2 + g^2C_x^2 - 2g\rho_{yx}C_yC_x\right] \tag{1.8}$$

where $C_y = \frac{S_y}{\bar{Y}}$, $C_x = \frac{S_x}{\bar{X}}$, $\rho_{yx} = \frac{S_{yx}}{S_y S_x}$, $\gamma = \frac{1}{n}$(ignoring $f = \frac{n}{N}$) , $S_y^2 = \frac{\sum_{i=1}^{N}(y_i - \bar{Y})^2}{N-1}$,

$S_x^2 = \frac{\sum_{i=1}^{N}(x_i - \bar{X})^2}{N-1}$, $S_{yx} = \frac{\sum_{i=1}^{N}(y_i - \bar{Y})(x_i - \bar{X})}{N-1}$,$\theta = \frac{a\bar{X}}{2(a\bar{X}+b)}$ and $g = \frac{mr}{N-mr}$

2. RATIO ESTIMATORS IN RANKED SET SAMPLING

In Ranked set sampling (RSS), m independent random sets are chosen, each of size m and units in each set are selected with equal probability and without replacement from the population. The members of each random set are ranked with respect to the characteristic of the study variable or auxiliary variable. Then, the smallest unit is selected from the first ordered set and the second smallest unit is selected from the second ordered set. By this way, this procedure is continued until the unit with the largest rank is chosen from the m^{th} set. This cycle may be repeated r times, so $mr\ (=n)$ units have been measured during this process.

When we rank on the auxiliary variable, let $(y_{[i]}, x_{(i)})$ denote the i^{th} judgment ordering for the study variable and i^{th} perfect ordering for the auxiliary variable in the i^{th} set, where $i = 1,2,3.........., m.$

Swami (1996) defined the ratio estimator for the population mean in ranked set sampling as

$$\bar{y}_{R,RSS} = \bar{y}_{[n]}\left(\frac{\bar{X}}{\bar{x}_{(n)}}\right), \tag{2.1}$$

where $\bar{y}_{[n]} = \frac{1}{n}\sum_{i=1}^{n} y_{[i]}$, $\bar{x}_{(n)} = \frac{1}{n}\sum_{i=1}^{n} x_{(i)}$ are the ranked set sample means for variables y and x respectively.

To the first degree of approximation the mean squared errors (MSE's) of the estimators $\bar{y}_{R,RSS}$ is given as

$$MSE(\bar{y}_{R,RSS}) = \bar{Y}^2\left[\theta\{C_y^2 + C_x^2 - 2\rho_{yx}C_yC_x\} - \{W_{y[i]} - W_{x(i)}\}^2\right] \tag{2.2}$$

3. MODIFIED EXPONENTIAL ESTIMATOR USING RANKED SET SAMPLING

Motivated by Singh Rajesh et al (2007), We proposed modified exponential estimator for $\bar{Y}$ using Ranked set sampling as

$$\bar{y}_{e,RSS} = \bar{y}_{[n]}\exp\left[\frac{(a\bar{X}+b)-(a\bar{x}_{(n)}+b)}{(a\bar{X}+b)+(a\bar{x}_{(n)}+b)}\right] \tag{3.1}$$

Here $\bar{y}_{[n]} = \frac{1}{n}\sum_{i=1}^{n} y_{[i]}$, $\bar{x}_{(n)} = \frac{1}{n}\sum_{i=1}^{n} x_{(i)}$ are the ranked set sample means for variables y and x respectively.

The Bias and MSE of $\bar{y}_{e,RSS}$ can be found as follows-

$$B(\bar{y}_{e,RSS}) = E(\bar{y}_{e,RSS}) - \bar{Y}$$

Here $\bar{y}_{e,RSS} = \bar{Y}(1+\varepsilon_0)\exp\left[\frac{a\bar{X} - a\bar{X}(1+\varepsilon_1)}{a\bar{X} + 2b + a\bar{X}(1+\varepsilon_1)}\right]$

$= \bar{Y}(1+\varepsilon_0)\exp\left[-\theta\varepsilon_1(1+\theta\varepsilon_1)^{-1}\right]$ where $\theta = \frac{a\bar{X}}{2(a\bar{X}+b)}$

Expanding the right hand side of eq. and retaining terms up to second power of ε's , we have

$$E(\bar{y}_{e,RSS}) = \bar{Y}E\left(1+\varepsilon_0 - \theta\varepsilon_1 + \theta^2\varepsilon_1^2 + \theta\varepsilon_0\varepsilon_1\right)$$

So that $B(\bar{y}_{e,RSS}) = \bar{Y}\left[\theta^2 E(\varepsilon_1) + \theta E(\varepsilon_0\varepsilon_1)\right]$, because $E(\varepsilon_0) = E(\varepsilon_1) = 0$

$$\Rightarrow B(\bar{y}_{e,RSS}) = \bar{Y}\left[\gamma\left\{\theta^2 C_x^2 + \theta\rho C_x C_y\right\} - \left\{\theta^2 W_{x(i)}^2 + \theta W_{yx(i)}\right\}\right] \tag{3.2}$$

Now $MSE(\bar{y}_{e,RSS}) = E(\bar{y}_{e,RSS} - \bar{Y})^2$

$$= \bar{Y}^2 E[\varepsilon_0 - \theta\varepsilon_1]^2$$

$$= \bar{Y}^2 E\left[\varepsilon_0^2 + \theta^2\varepsilon_1^2 - 2\theta\varepsilon_0\varepsilon_1\right]$$

$$= \bar{Y}^2\left[\gamma C_y^2 - W_{y[i]}^2 + \theta^2(\gamma C_x^2 - W_{x(i)}^2) - 2\theta\left(\gamma\rho_{yx}C_y C_x - W_{yx(i)}\right)\right]$$

$$\Rightarrow MSE(\bar{y}_{e,RSS}) = \bar{Y}^2\left[\gamma\{C_y^2 + \theta^2 C_x^2 - 2\theta\rho_{yx}C_y C_x\} - \{W_{y[i]} - \theta W_{x(i)}\}^2\right] \tag{3.3}$$

The following table shows some other estimators of the population mean which can be obtained by putting different values of constants a and b.

Estimator	Different values of	
	a	**b**
$\bar{y}_{e1,RSS}=\bar{y}_{[n]}$	0	0
$\bar{y}_{e2,RSS}=\bar{y}_{[n]}\exp\left[\frac{\bar{X}-\bar{x}_{(n)}}{\bar{X}+\bar{x}_{(n)}}\right]$	1	1
$\bar{y}_{e3,RSS}=\bar{y}_{[n]}\exp\left[\frac{\bar{X}-\bar{x}_{(n)}}{\bar{X}+\bar{x}_{(n)}+2\beta_2(x)}\right]$	1	$\beta_2(x)$
$\bar{y}_{e4,RSS}=\bar{y}_{[n]}\exp\left[\frac{\bar{X}-\bar{x}_{(n)}}{\bar{X}+\bar{x}_{(n)}+2C_x}\right]$	1	C_x
$\bar{y}_{e5,RSS}=\bar{y}_{[n]}\exp\left[\frac{\bar{X}-\bar{x}_{(n)}}{\bar{X}+\bar{x}_{(n)}+2\rho}\right]$	1	ρ
$\bar{y}_{e6,RSS}=\bar{y}_{[n]}\exp\left[\frac{\beta_2(x)(\bar{X}-\bar{x}_{(n)})}{\beta_2(x)(\bar{X}+\bar{x}_{(n)})+2C_x}\right]$	$\beta_2(x)$	C_x
$\bar{y}_{e7,RSS}=\bar{y}_{[n]}\exp\left[\frac{C_x(\bar{X}-\bar{x}_{(n)})}{C_x(\bar{X}+\bar{x}_{(n)})+2\beta_2(x)}\right]$	C_x	$\beta_2(x)$
$\bar{y}_{e8,RSS}=\bar{y}_{[n]}\exp\left[\frac{C_x(\bar{X}-\bar{x}_{(n)})}{C_x(\bar{X}+\bar{x}_{(n)})+2\rho}\right]$	C_x	ρ
$\bar{y}_{e9,RSS}=\bar{y}_{[n]}\exp\left[\frac{\rho(\bar{X}-\bar{x}_{(n)})}{\rho(\bar{X}+\bar{x}_{(n)})+2C_x}\right]$	ρ	C_x
$\bar{y}_{e10,RSS}=\bar{y}_{[n]}\exp\left[\frac{\beta_2(x)(\bar{X}-\bar{x}_{(n)})}{\beta_2(x)(\bar{X}+\bar{x}_{(n)})+2\rho}\right]$	$\beta_2(x)$	ρ
$\bar{y}_{e11,RSS}=\bar{y}_{[n]}\exp\left[\frac{\rho(\bar{X}-\bar{x}_{(n)})}{\rho(\bar{X}+\bar{x}_{(n)})+2\beta_2(x)}\right]$	ρ	$\beta_2(x)$

It is cleared that bias and MSE of the above estimators given in the table can be obtained by substituting the values of a and b in (3.2) and (3.3) respectively.

4. MODIFIED DUAL RATIO ESTIMATOR IN RANKED SET SAMPLING

Motivated by Srivenkataramana and Tarcy (1980), we propose dual ratio-type estimator for using Ranked set sampling as

$$\bar{y}_{nsu,RSS}=\bar{y}_{(n)}\left[\frac{N\bar{X}-mr\bar{x}_{(n)}}{(N-mr)\bar{X}}\right] \tag{4.1}$$

or $\bar{y}_{nsu,RSS}=\bar{y}_{(n)}\left[\frac{\bar{x}^{*}_{(n)}}{\bar{X}}\right]$, where $\bar{x}^{*}_{(n)}=\frac{N\bar{X}-mr\bar{x}_{(n)}}{(N-mr)}$

The Bias and MSE of $\overline{y}_{nsu,RSS}$ can be found as follows

$$B(\overline{y}_{nsu,RSS}) = E(\overline{y}_{nsu,RSS}) - \overline{Y}$$

$$\overline{y}_{nsu,RSS} = \overline{Y}(1+\varepsilon_0)\left[\frac{N\overline{X} - mr\overline{X}(1+\varepsilon_1)}{(N-mr)\overline{X}}\right]$$

$$= \overline{Y}(1+\varepsilon_0)\left(1 - \frac{mr}{N-mr}\varepsilon_1\right)$$

$$= \overline{Y}\left(1+\varepsilon_0 - \frac{mr}{N-mr}\varepsilon_1 - \frac{mr}{N-mr}\varepsilon_0\varepsilon_1\right)$$

Taking expected values on both sides, we have

$$E(\overline{y}_{nsu,RSS}) = \overline{Y}\left[1 + E(\varepsilon_0) - \frac{mr}{N-mr}E(\varepsilon_1) - \frac{mr}{N-mr}E(\varepsilon_0\varepsilon_1)\right]$$

$$= \overline{Y} - \frac{mr}{N-mr}\overline{Y}\ E(\varepsilon_0\varepsilon_1)$$

Now $B(\overline{y}_{nsu,RSS}) = E(\overline{y}_{nsu,RSS})$

$$= -\frac{1}{N-mr}\frac{1}{\overline{X}}\left[S_{yx} - \frac{1}{m}\sum_{i=1}^{m}\tau_{yx(i)}\right]$$

$$= -\frac{mr}{N-mr}\overline{Y}\{\gamma\rho_{xy}C_xC_y - W_{yx(i)}\}$$

$$\Rightarrow B(\overline{y}_{nsu,RSS}) = -g\overline{Y}\{\gamma\rho_{xy}C_xC_y - W_{yx(i)}\} \quad , \quad \text{where} \quad g = \frac{mr}{N-mr} \qquad (4.2)$$

Now $MSE(\overline{y}_{nsu,RSS}) = E(\overline{y}_{nsu,RSS} - \overline{Y})^2$

$$= E\left[\overline{Y}\left(1+\varepsilon_0 - \frac{mr}{N-mr}\varepsilon_1 - \frac{mr}{N-mr}\varepsilon_0\varepsilon_1\right) - \overline{Y}\right]^2$$

$$= \overline{Y}^2 E\left[\varepsilon_0 - \frac{mr}{N-mr}\varepsilon_1\right]^2$$

$$=\bar{Y}^2\left[\left\{\gamma C_y^2 - W_{y(i)}^2\right\}+\left(\frac{mr}{N-mr}\right)^2\left\{\gamma C_x^2 - W_{x(i)}^2\right\}-2\left(\frac{mr}{N-mr}\right)\left\{\gamma\rho C_y C_x - W_{yx(i)}\right\}\right]$$

$$=\bar{Y}^2\left[\gamma\left\{C_y^2 + g^2 C_x^2 - 2g\rho_{xy}C_x C_y\right\}-\left\{W_{y(i)}^2 + g^2 W_{x(i)}^2 - 2gW_{yx(i)}\right\}\right]$$

$$\Rightarrow \text{MSE}(\bar{y}_{nsu,RSS}) = \bar{Y}^2\left[\gamma\{C_y^2 + g^2 C_x^2 - 2g\rho_{yx}C_y C_x\} - \{W_{y[i]} - gW_{x(i)}\}^2\right] \tag{4.3}$$

5. EFFICIENCY COMPARISON

On comparing (1.6) and (1.8) with (3.3) and (4.3) respectively, we obtain

$$MSE(\bar{y}_e) - MSE(\bar{y}_{e,RSS}) = A_1 \geq 0, \text{ where } A_1 = \left[W_{y[i]} - \theta W_{x(i)}\right]^2$$

$$\Rightarrow MSE(\bar{y}_{e,RSS}) \leq MSE(\bar{y}_e)$$

$$MSE(\bar{y}_{nsu}) - MSE(\bar{y}_{nsu,RSS}) = A_2 \geq 0, \text{ where } A_2 = \left[W_{y[i]} - gW_{x(i)}\right]^2$$

$$\Rightarrow MSE(\bar{y}_{nsu,RSS}) \leq MSE(\bar{y}_{nsu})$$

It is easily seen that the MSE of the suggested estimators given in (3.3) and (4.3) are always smaller than the estimator given in (1.6) and (1.8) respectively, because A_1 and A_2 all are non-negative values. As a result, show that the proposed estimators $\bar{y}_{e,RSS}$ and $\bar{y}_{nsu,RSS}$ for the population mean using RSS are more efficient than the usual estimators $\bar{y}_e$ and $\bar{y}_{nsu}$ respectively.

Now Comparison between (2.2) and (3.3), we obtain the estimator $\bar{y}_{nsu,RSS}$ will be more efficient than the ratio estimator $\bar{y}_{r,RSS}$ if

$$\text{MSE}\left(\bar{y}_{nsu,RSS}\right) < MSE\left(\bar{y}_{r,RSS}\right)$$

$$\bar{Y}^2\left[\gamma\left\{C_y^2 + g^2 C_x^2 - 2g\rho C_y C_x\right\}-\left\{W_{y(i)}^2 + g^2 W_{x(i)}^2 - 2gW_{yx(i)}\right\}\right]$$

$$< \bar{Y}^2\left[\gamma\left\{C_y^2 + C_x^2 - 2\rho C_y C_x\right\}-\left\{W_{y(i)}^2 + W_{x(i)}^2 - 2W_{yx(i)}\right\}\right]$$

$$\Rightarrow \left\{\left(g^2-1\right)\gamma C_x^2 - 2(g-1)\gamma\rho C_y C_x\right\}-\left\{\left(g^2-1\right)W_{x(i)}^2 - 2(g-1)W_{yx(i)}\right\}<0$$

$$\Rightarrow \left(g^2-1\right)\left\{\gamma C_x^2 - W_{x(i)}^2\right\}-2(g-1)\left\{\rho\gamma C_y C_x - W_{yx(i)}\right\}<0$$

$$\Rightarrow (g-1)\left[(g+1)\left\{\gamma C_x^2 - W_{x(i)}^2\right\} - 2\left\{\gamma\rho C_y C_x - W_{yx(i)}\right\}\right] < 0 \tag{5.1}$$

Now there are two cases -

Case 1: The inequality (5.1) will be satisfied if

$$g-1<0 \quad \text{and} \quad \left[(g+1)\left\{\gamma C_x^2 - W_{x(i)}^2\right\} - 2\left\{\gamma\rho C_y C_x - W_{yx(i)}\right\}\right] > 0$$

or $\quad \dfrac{mr}{N-mr} - 1 < 0 \quad$ and $\quad (g+1)\left\{\gamma C_x^2 - W_{x(i)}^2\right\} > 2\left\{\gamma\rho C_y C_x - W_{yx(i)}\right\}$

or $\quad \dfrac{mr - N + mr}{N-mr} < 0 \quad$ and $\quad \dfrac{\gamma\rho C_y C_x - W_{yx(i)}}{\gamma C_x^2 - W_{x(i)}^2} < \dfrac{(g+1)}{2}$

or $\quad mr < \dfrac{N}{2} \quad$ and $\quad \dfrac{\gamma\rho C_y C_x - W_{yx(i)}}{\gamma C_x^2 - W_{x(i)}^2} < \dfrac{N}{2(N-mr)}$

or $\quad mr < \dfrac{N}{2} \quad$ and $\quad \dfrac{Cov\left(\bar{x}_{(n)}, \bar{y}_{[n]}\right)/\bar{X}\bar{Y}}{V\left(\bar{x}_{(n)}\right)/\bar{X}^2} < \dfrac{N}{2(N-mr)} \quad (5.2)$

As in the case of SRS, it is clear that to the first order of approximation the RSS estimators are unbiased, using $Cov(\bar{x}_{(n)}, \bar{y}_{[n]}) = \beta V\left(\bar{x}_{(n)}\right)$ in (5.2), we obtain

$$\frac{\beta Var\left(\bar{x}_{(n)}\right)}{Var\left(\bar{x}_{(n)}\right)} \frac{\bar{X}}{\bar{Y}} < \frac{N}{N-mr}$$

and $\rho xy \dfrac{C_y}{C_x} < \dfrac{N}{2(N-mr)}$

for $C_y \cong C_x$ we have $mr < \dfrac{N}{2}$ and $\rho xy < \dfrac{N}{2(N-mr)}$

This condition holds in practice. For example, if N=100 and $mr = 15$ then ρxy is supposed to be less than 0.60.

Case 2: The inequality (3) will be satisfied if

$$(g-1) > 0 \quad \text{and} \quad \left[(g+1)\left\{C_x^2 - W_{x(i)}^2\right\} - 2\left\{\gamma\rho C_y C_x - W_{yx(i)}^2\right\}\right] < 0$$

Here for $C_y \cong C_x$, we have $n > \dfrac{N}{2}$ & $\rho_{xy} > \dfrac{N}{2(N-mr)}$

This condition will not hold in practice. For example, if N=100 and $mr = 70$ then ρxy is supposed to be greater than 1.667, which is not possible.

REFERENCES

1. *Bahl, S. and Tuteja, R.K. (1991). Ratio and Product type exponential estimator, Information and Optimization sciences, Vol.XII, I, 159-163.*
2. *Cochran, W.G. (1940). Some properties of estimators based on sampling scheme with varying probabilities. Austral. J. Statist., 17, 22-28.*
3. *Kadilar, C. and Cingi, H. (2006(b)).Improvement in estimating the population mean in simple random sampling. Applied Mathematics Letters, 19, 75-79.*
4. *Kadilar, C.,Unyazici, Y. and Cingi H.(2009), Ratio estimator for the population mean using ranked set sampling, Stat. papers,50,301-309.*
5. *Khoshnevisan, M. Singh, R., Chauhan, P., Sawan, N. and Smarandache, F. (2007): A general family of estimators for estimating population means using known value of some population parameter(s). Far East journal of theoretical statistics, 22(2), 181-191.*
6. *McIntyre, G.A.(1952), A method of unbiased selective sampling using ranked sets, Australian Journal of Agricultural Research, 3, 385-390.*
7. *Mehta (Ranka), Nitu and Mandowara, V.L. (2016). A Modified Ratio-Cum-Product Estimator of Finite Population Mean Using Ranked Set Sampling. Communication and Statistics-Theory and Methods.Vol.45 (2), 267-276.*
8. *Prasad,B. (1989), Some improved ratio type estimators of population mean and ratio in finite population sample surveys. Commun Stat Theory Methods, 18,379–392.*
9. *Singh R, Chauhan P. and Sawan N.(2007). Improvement in Estimating the Population Mean using Exponential Estimator in Simple Random Sampling. Renaissance High Press, 33-40.*
10. *Samawi, H.M., and Muttlak, H.A. (1996): Estimation of ratio using rank set sampling. The Biometrical Journal, 38, 753-764.*
11. *Singh, M.P. (1967). Ratio-cum-product method of estimation, Metrika, 12, 34-42.*
12. *Sisodia, B.V.S. and Dwivedi, V.K. (1981). A modified ratio estimator using coefficient of variation of auxiliary variable. Journal of Indian Society of Agricultural Statistics, 33, 13-18.*
13. *Srivenkatramana, T. (1980). A dual to ratio estimator in sample surveys, Biometrika, 67(1), 199–204.*
14. *Tailor, R. and Sharma, B. (2009). A Modified Ratio-Cum-Product Estimator of Finite Population Mean Using Known Coefficient of Variation and Coefficient of Kurtosis. Statistics in Transition-new series, Vol.10 (1), 15-24.*

APPENDIX

To obtain bias and MSE of $\overline{y}_{e,RSS}$, we put $\overline{y}_{[n]} = \overline{Y}(1+\varepsilon_0)$ and $\overline{x}_{(n)} = \overline{X}(1+\varepsilon_1)$ so that $E(\varepsilon_0) = E(\varepsilon_1) = 0$, and therefore,

$$V(\varepsilon_0) = E(\varepsilon_0^2) = \frac{V(\overline{y}_{[n]})}{\overline{Y}^2}$$

$$= \frac{1}{mr}\frac{1}{\overline{Y}^2}\left[S_y^2 - \frac{1}{m}\sum_{i=1}^{m}\tau_{y[i]}^2\right] = \left[\gamma C_y^2 - W_{y[i]}^2\right]$$

similarly, $V(\varepsilon_1) = E(\varepsilon_1^2) = \left[\gamma C_x^2 - W_{x(i)}^2\right]$

and $Cov(\varepsilon_0, \varepsilon_1) = E(\varepsilon_0, \varepsilon_1) = \dfrac{Cov\left(\overline{y}_{[n]}, \overline{x}_{(n)}\right)}{\overline{X}\,\overline{Y}}$

$$= \frac{1}{\overline{X}\,\overline{Y}}\frac{1}{mr}\left[S_{yx} - \frac{1}{m}\sum_{i=1}^{m}\tau_{yx(i)}\right] = \left[\gamma\rho_{yx}C_y C_x - W_{yx(i)}\right]$$

where $\gamma = \dfrac{1}{mr}$, $\varepsilon_0 = \dfrac{\overline{y}_{[n]} - \overline{Y}}{\overline{Y}}$, $\varepsilon_1 = \dfrac{\overline{x}_{(n)} - \overline{X}}{\overline{X}}$, $C_y^2 = \dfrac{S_y^2}{\overline{Y}^2}$, $C_x^2 = \dfrac{S_x^2}{\overline{X}^2}$, $C_{yx} = \dfrac{Syx}{\overline{X}\,\overline{Y}} = \rho_{yx}C_y C_x$

$W_{x(i)}^2 = \dfrac{1}{m^2 r}\dfrac{1}{\overline{X}^2}\sum_{i=1}^{m}\tau_{x(i)}^2$, $W_{y[i]}^2 = \dfrac{1}{m^2 r}\dfrac{1}{\overline{Y}^2}\sum_{i=1}^{m}\tau_{y[i]}^2$ and $W_{yx(i)} = \dfrac{1}{m^2 r}\dfrac{1}{\overline{X}\,\overline{Y}}\sum_{i=1}^{m}\tau_{yx(i)}$. Here we would like to remind that $\tau_{x(i)} = \mu_{x(i)} - \overline{X}$, $\tau_{y[i]} = \mu_{y[i]} - \overline{Y}$ and $\tau_{yx(i)} = (\mu_{x(i)} - \overline{X})\,(\mu_{y[i]} - \overline{Y})$.

Further to validate first degree of approximation, we assume that the sample size is large enough to get $|\varepsilon_0|$ and $|\varepsilon_1|$ as small so that the terms involving ε_0 and or ε_1 in a degree greater than two will be negligible.